Optimal Delegation and Limited Awareness*

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Abstract

We study the delegation problem between a principal and an agent, who not only has better information about the performance of the different available actions but also has superior awareness of the set of actions that are actually feasible. The agent decides which of the feasible actions to reveal and which ones to hide. We show that it is optimal for the agent to make the principal aware of extremes options, while leaving him unaware of intermediate ones. Allowing the principal and agent to renegotiate highlights interesting, new effects of limited awareness on strategic information transmission.

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1 Introduction

In many situations economic agents need to rely on the advice of experts whose preferences are not perfectly aligned with their own. Headquarters depend on division managers who

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have superior information about the profitability of available projects but also a desire to attract additional resources to their own division, voters rely on politicians whose preferences may reflect a political bias or the interest of certain lobbies, financial investors seek advice from non-neutral financial professionals with a better understanding of the risks and returns of the available portfolios. The tension underlying these situations has been formalized in the delegation model - first introduced by Holmström (1977) - where an uninformed principal specifies a set of permissible actions to the informed agent and contingent transfers are infeasible. In most of the described situations, however, it seems plausible that the informed party not only has a better understanding of what the most suitable action is but also of the set of actions that are actually available, e.g. the set of feasible projects, available policies, realizable investments, etc.

This paper studies the implications of such asymmetry by incorporating unawareness into the canonical delegation model. We consider the problem of a principal (she) who needs to take an action and delegates the task to an agent (he). The agent has private information about the payoffs of each of the available actions and the principal's problem is to determine a set of actions from which the agent can choose (see for example Alonso and Matouschek, 2008). We depart from this traditional framework of optimal delegation by considering a situation where the agent not only has private information about the suitability of the different actions, but also about the set of actions that are actually available to the principal. This second dimension of asymmetry is captured by the assumption that the principal is only *partially aware of the feasible actions*. Before the delegation stage the agent has the possibility to enrich the principal's awareness by revealing additional actions. We are interested in the questions of whether the agent expands the principal's awareness, which actions the agent reveals, and what the properties of the realized actions are.

We address these questions in an environment with a continuum of states and a continuum of actions, some of which the principal is aware of. Contingent on the state of the world the agent always wants to take a higher action than the principal, which creates a tension with regard to how much flexibility the agent to grant. In the benchmark case of full awareness the optimal delegation set for the principal is an interval: the principal effectively imposes an upper cap below which the agent is free to choose. In the case of partial awareness, the principal's delegation choice depends on the set of actions she is aware of. Anticipating this, the agent chooses which actions to reveal and which ones to hide. Despite the fact that we impose very little structure on the principal's initial awareness set, we are able to obtain a clean description of the equilibrium: our main result shows that generically the agent leaves the principal unaware of an interval of intermediate options around the optimal upper cap under full awareness.[1] In other words, the agent makes the principal *aware of actions at the extremes*. The awareness gap is chosen in a way such that the principal - who still cares about the agent's information - finds it optimal to permit actions at both extremes. Thus, by leaving the principal unaware of intermediate options, the agent is able to select actions that would be precluded if the principal was fully aware.

To prove the result we proceed in two steps. First, we characterize the optimal delegation set for an arbitrary awareness set of the principal. Here we extend the characterization result of the existing literature on optimal delegation to the case where the set of actions available to the principal must not be an interval. We show that, within the constraints of her limited awareness, the principal chooses the closest approximation of the optimal delegation interval under full awareness. The optimal upper bound of this approximation is simply the action in the principal's awareness closest to the optimal cap under full awareness. In the second step, we turn to the question of which actions the agent optimally reveals to the principal. The agent must chose over subsets of feasible actions, taking into account the principal's initial awareness set. Solving this optimization problem becomes tractable due to our characterization of the optimal delegation set for a given awareness set: the agent can induce the principal to permit an action strictly higher than the optimal cap under full awareness if and only if the principal remains unaware of an interval of intermediate actions around the cap. We then show that, whenever it is feasible with respect to the principal's initial awareness set, the agent can strictly gain by leaving the principal unaware of such

[1]More precisely, this always happens unless the principal is exactly aware of the action at the optimal upper bound under full awareness.

interval. The larger the bias of the agent is, the greater is the size of the interval.

When the agent does not reveal all actions to the principal, there are states of the world where the utility of the principal and the agent can be improved by taking an intermediate action which has not been revealed. This gives rise to the question of what would happen if, after learning the state of the world, the agent can reveal additional actions to the principal. We address this question by extending the baseline model to allow for renegotiation. We show that there is an equilibrium with the following features: before learning the state of the world, the agent leaves the principal unaware of an interval of intermediate actions, which is weakly larger than in the case without renegotiation. When the agent learns that his preferred action falls into the interval, he reveals his preferred action to the principal and proposes to take that action instead. The principal infers that the agent prefers the newly revealed action over the initial delegation set but cannot conceive of alternative actions the agent would have revealed in other states of the world. As a result, the principal only partially learns about the realized state, despite the agent's action being fully revealing from the modeller's viewpoint. We then show that, given the principal's partial information, she optimally permits the agent to implement the newly revealed action.

The described equilibrium highlights two interesting features of games with limited awareness. First, the possibility of renegotiation leads the agent to reveal fewer actions in the beginning of the game. Unawareness is not reversible, which means that revealing actions at the outset of the game shrinks the collection of awareness sets the agent can induce at a later stage. Secondly, in the renegotiation stage principal and agent play a cheap talk game, yet the agent's equilibrium action is strictly increasing on an interval of states. As we mentioned above, this is possible because the principal's limited awareness restricts the extent to which she infers information from the agent's equilibrium action. Limited awareness can thus lead to rather different equilibrium outcomes in games of strategic information transmission compared to the benchmark model with full awareness.

Finally, we discuss the robustness of our results. While the baseline model assumes that the set of feasible actions is an interval, we show that the characterization can be extended to the case where the set of feasible actions is an arbitrary subset of the reals. We also discuss more general utility functions and specifications of the bias. Furthermore, we show how our main result extends to the case where the agent faces uncertainty about the principal's initial awareness and, lastly, we discuss the possibility of allowing for message contingent transfers.

Related Literature: This paper is first of all related to the literature on optimal delegation. Starting with Holmström (1984), who first defines the delegation problem and provides conditions for the existence of its solution, this literature, which includes Melamud and Shibano (1991), Martimort and Semenov (2006), Alonso and Matouschek (2008), Armstrong and Vickers (2010) and Amador and Bagwell (2013), Halac and Yared (2017), and others, studies optimal delegation problems in environments of increasing generality. None of them consider limited awareness in this framework. Szalay (2005) considers a delegation problem where the agent has costly access to valuable information. He shows that - in order to motivate information acquisition - it might be optimal to remove intermediate actions from the delegation set, which are by assumption optimal when the information is poor.

Furthermore, our work is related to a relatively small literature on contract theory and unawareness. The application of the concept of unawareness to contracting problems is still at its beginnings. In contrast to our setting, existing work considers contracting problems where contingent transfers are feasible and where the agent is unaware - either of possible actions (Von Thadden and Zhao, 2012 and 2014) or of possible states (Zhao, 2011; Filiz-Ozbay, 2012; Auster, 2013) - while the principal is fully aware.

In a companion paper, Auster and Pavoni (2018), we apply our model to a financial market setting, considering a market with multiple fully aware brokers (agents) and a continuum of partially aware investors (principals). We study the effects of competition and investor heterogeneity on the market outcome. Self-reported data from customers in the Italian retail investment sector support the key predictions of the model: the menus offered to less knowledgable investors contain fewer products, most of them nevertheless perceived to be at the extremes.

The paper is organized as follows. The next section presents the delegation model with limited awareness. In Section 3 we derive the equilibrium awareness and delegation set. Section 4 extends the baseline model to allow for renegotiation. Finally, in Section 5 we discuss a number of interesting robustness checks and in Section 6 we conclude.

2 Environment

There is a principal and an agent. The agent has access to a set of actions, the payoffs to which depend on the state of the world. The principal is only aware of a subset of those actions, denoted by $Y^P \subseteq Y^A$.[2] We think of Y^A as a general subset of $\mathbb{R}$, for instance a finite collection of points. For expositional purposes, however, we will initially assume that the underlying set of actions is an interval, i.e. $Y^A = [y_{min}, y_{max}]$. This will simplify notation considerably and we discuss the extension to more general sets in Section 5. We assume the set Y^P is closed but impose no assumptions otherwise.

Let $\Theta = [0, 1]$ be the set of states and let $F(\theta)$ denote the cumulative distribution function on Θ, assumed to be twice differentiable on the support.[3] Both the principal and the agent have von-Neumann-Morgenstern utility functions that take the quadratic form

$$u(y, \theta) = -(y - \theta)^2 \quad \text{and} \quad v(y, \theta) = -(y - (\theta - \beta))^2.$$

The agent's preferred policy is $y = \theta$, while the principal's preferred policy is $y = \theta - \beta$. We assume $\beta > 0$, hence the agent has an upward bias of size β.

The agent is privately informed about the state of the world θ. We rule out monetary transfers and assume that the agent's participation constraint is always satisfied. The contracting problem of the principal then reduces to the decision over the set of actions from which the agent can choose.[4] However, the principal's unawareness restricts the language

[2]Hence, unawareness in our framework does not take the form of unforeseen contingencies but concerns the set of available actions: while the principal knows the state space, she has an incomplete understanding of the set of actions that are feasible. Karni and Vierø (2015 and 2017) formalize this idea in a decision theoretic model that not only allows for unawareness of contingencies and outcomes but also of acts.

[3]For $\theta = 0$ and $\theta = 1$, this condition holds for, respectively, the right and left derivative.

[4]Formally, the principal commits to a mechanism that specifies the action which will be implemented as a function of the agent's message. Alonso and Matouschek (2008) show that this contracting problem is equivalent to delegating a set of actions $D \subseteq Y^A$ from which the principal can choose freely after observing

with which she can write a contract. In particular, we assume that the principal can only include actions into the contract that she can name explicitly. This implies that her delegation set must be a subset of her awareness set.[5] Thus, the larger the principal's awareness set is, the richer is the set of contracts she can write.

Before the principal makes her delegation choice and the agent observes the state of the world, the agent can make the principal aware of additional actions.[6] The principal fully understands the options that are revealed to her and accordingly updates her awareness to the union of whatever she knew initially and what the agent reveals. Given her updated awareness, the principal determines a delegation set. Finally the agent learns the state of the world and chooses a action from those permitted by the principal. The timing of the game can be summarized as follows:

1. The principal's initial awareness Y^P is realized and observed by all parties.
2. The agent reveals a set of actions $X \subseteq Y^A$ and the principal updates her awareness to $Y \equiv Y^P \cup X$.
3. Given Y, the principal chooses a delegation set $D \in \mathcal{D}(Y)$, where $\mathcal{D}(Y)$ is the collection of closed subsets of Y.[7]
4. The agent observes the state of world θ and chooses an action from set D.
5. Payoffs are realized.

At this stage we do not need to make any explicit assumption on whether or not the principal is aware of her unawareness. The principal might take the world at face value or she might understand that there exist actions outside her awareness. Since she cannot include such actions in the delegation set, awareness of their possible existence neither affects her

the state of the world. Their argument continues to hold in our setting. There are thus two possible interpretations: after observing the state of the world, the agent might directly choose an action or make a recommendation which the principal has committed to follow.

[5] Alternatively, the principal might specify those actions that the agent is *not* allowed to take. It can be shown that in this case the agent will never have incentives to reveal any actions to the principal.

[6] We will discuss an alternative timing in Section 4.

[7] As discussed in Alonso and Matouschek (2008), the restriction to closed sets is without loss of generality.

expected payoff nor optimization problem.[8] As well, within the constraints of her awareness, the principal is perfectly rational: she anticipates correctly the expected payoff associated to each feasible delegation set and will not be surprised ex-post.

Formally the game between principal and agent can be represented by a family of partially ordered subjective game trees (see Feinberg (2012) and Heifetz et al. (2013)). Such family includes the modeler's view of the objectively feasible paths of play but also the feasible paths of play as subjectively viewed by some players, or as the frame of mind attributed to a player by other players or by the same player at a later stage of the game. Defining an equilibrium in this environment then requires specifying a collection of strategy profiles and belief systems for each member of the family of game trees. Since the principal's perception of the game can change over time, a strategy should not be viewed as an ex ante plan of action but as a list of answers to the the questions of what each player would do at an information set associated to a particular frame of mind (see Heifetz et al. 2013). While we provide a more detailed description of the formalization for our environment in the Online Appendix, Figure 1 shows an example of the principal's subjective game tree induced by the announcement of the agent.

3 Equilibrium Analysis

We will now proceed with the analysis of the awareness and delegation sets that obtain in equilibrium. Our equilibrium notion is Perfect Bayesian, suitably extended to the class of extensive form games with unawareness (see Feinberg, 2012, Definition 16). We will start our analysis by first describing the benchmark case of full awareness and then turn to the subject of our interest: the case of partial awareness. Before entering the equilibrium analysis, it is useful to mention that optimal awareness sets and optimal delegation sets will typically not be unique since different awareness sets may induce the same delegation set and different

[8] In a dynamic environment, where the principal has ways to expand her awareness set (for example, by using a costly technology or by sampling other agents), the initial awareness of being partially aware and the confidence about the value of new information might matter (Karni and Vierø, 2017).

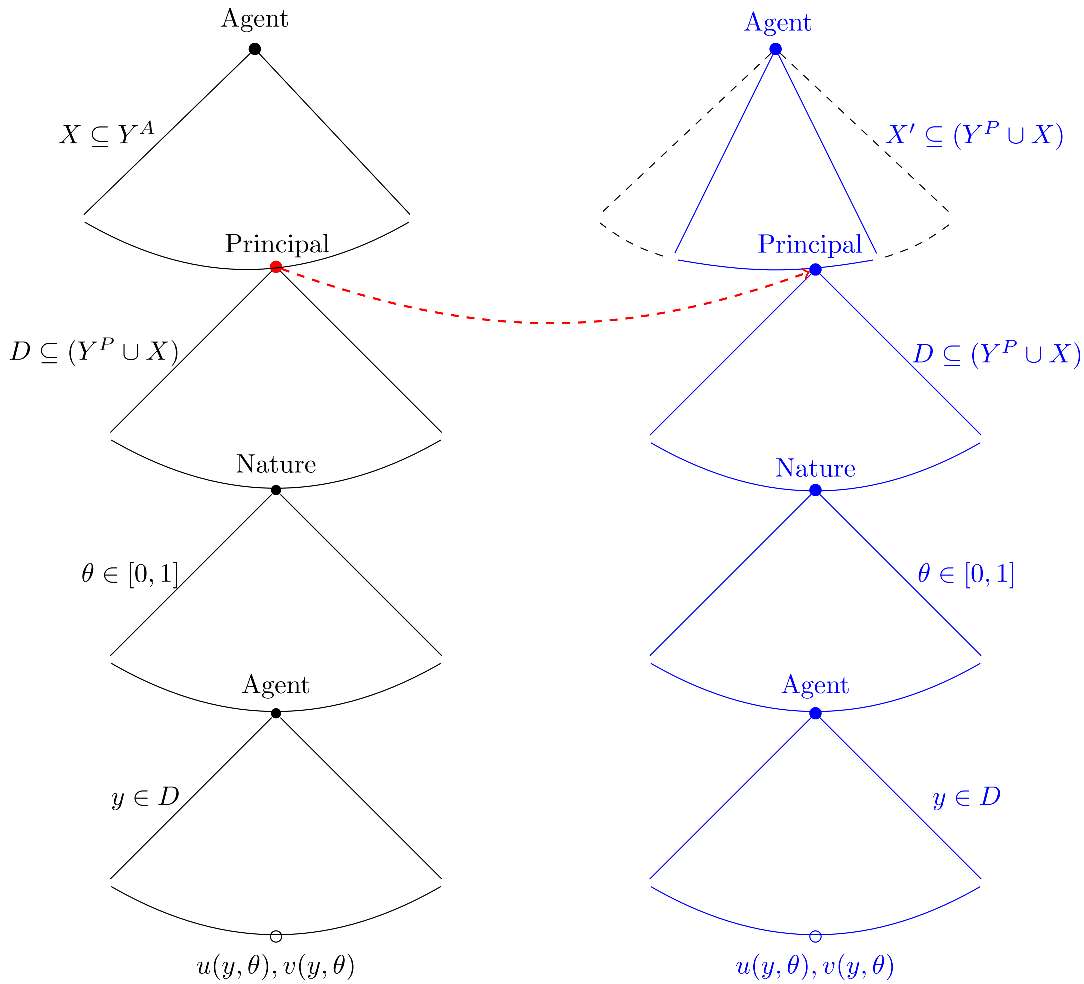

Figure 1: The left side shows the game tree as the agent perceives it. The right side shows the principal's perception of the game, induced by the agent's choice in the first stage (red dot)

delegation sets may induce the same implemented actions for each state of the world. In what follows, we will assume that, if the principal is indifferent between two delegation sets D and D' such that $D' \subset D$, she chooses the larger set D. Similarly, if the agent is indifferent between two revelation strategies that yield awareness sets Y and Y' such that $Y' \subset Y$, we will assume that he expands the principal's awareness to Y. That is, we will consider the sets that yield maximal awareness and maximal discretion.[9]

Throughout the analysis, we will adopt a couple of regularity conditions on the distribution that are common in the delegation literature. Furthermore, we will assume that in each state of the world both the principal's and the agent's ideal actions are available.

Assumption 1. $f'(\theta)\beta + f(\theta) > 0$ *for all* $\theta \in (0,1)$; *and* $\mathbb{E}[\theta - \beta] > 0$.[10]

Assumption 2. $y_{min} < -\beta$ *and* $y_{max} > 1$.

3.1 Full Awareness

For the specification $Y^P = Y^A$, the existing literature shows that if the density function $f(\theta) \equiv F'(\theta)$ satisfies the first regularity condition in Assumption 1, the optimal delegation set is an interval of the form $[y_{min}, \hat{y}]$ (Martimort and Semenov, 2006, and Alonso and Matouschek, 2008). The second condition in Assumption 1 guarantees that there exists some $\hat{y} > 0$ that solves[11]

$$\hat{y} = \mathbb{E}[\theta - \beta | \theta \geq \hat{y}]. \tag{1}$$

In that case, the agent chooses his preferred action $y = \theta$ for all $\theta < \hat{y}$ and the action $\hat{y}$ in all remaining states. Moreover, under the same assumptions, we can show that $\hat{y}$ decreases with the bias β.

[9] With this restriction we select one of multiple payoff equivalent equilibria. Otherwise it is not important.

[10] All expectations are taken with respect to F.

[11] If instead $\mathbb{E}[\theta - \beta] < 0$, the optimal delegation set is $[y_{min}, \mathbb{E}[\theta - \beta]]$. In this case, however, the agent will choose the upper bound of the set for all θ, so it is effectively the singleton $\{\mathbb{E}[\theta - \beta]\}$ and delegation has no value.

Proposition 1. *Under Assumptions 1 and 2 the maximizing solution to the optimal delegation problem with full awareness is an interval of the form* $[y_{min}, \hat{y}]$ *where the (unique) upper bound* $\hat{y} \in (0, 1)$ *solves equation* (1)*.*
Moreover, under the same assumptions, if we let $\hat{y}(\beta)$ *be the cap in the optimal delegation set when principal's preferences parameter is* $\beta \in (0, \mathbb{E}[\theta])$ *then* $\hat{y}(\cdot)$ *is decreasing and continuously differentiable.*

Proof. See Appendix A.1. □

For a graphical illustration of why the optimal delegation set takes this form see Figure 6 in Appendix A.1.

3.2 Partial Awareness: Main Result

Our main result shows that, maintaining the regularity condition on the state distribution, it is strictly optimal for the agent to leave the principal partially unaware if and only if the principal is initially unaware of the action at the optimal threshold under full awareness, $\hat{y}$. In that case, the agent optimally reveals actions at the extremes but leaves the principal unaware of intermediate options.

Proposition 2. *Let Assumptions 1 and 2 be satisfied.*

- *If* $\hat{y} \in Y^P$*, the principal becomes fully aware and the optimal delegation set is* $[y_{min}, \hat{y}]$*.*
- *If* $\hat{y} \notin Y^P$*, the principal remains unaware of actions in* $(\hat{y} - \Delta, \hat{y} + \Delta)$ *for some* $\Delta > 0$ *and the optimal delegation set is* $[y_{min}, \hat{y} - \Delta] \cup \{\hat{y} + \Delta\}$*.*

Proposition 2 shows that whether the principal is made aware of all actions by the agent is determined only by her awareness of $\hat{y}$, the optimal cap under full awareness. If she is unaware of $\hat{y}$, the agent optimally leaves the principal unaware of an interval of actions around $\hat{y}$. As we will show, this makes it optimal for the principal to choose a delegation set that includes a action to the right of $\hat{y}$. By leaving the principal unaware of intermediate actions, the agent thus incentivizes the principal to permit actions that the agent is biased towards and that would be precluded under full awareness. As a result, the equilibrium delegation set is no longer an interval, illustrated in Figure 5.

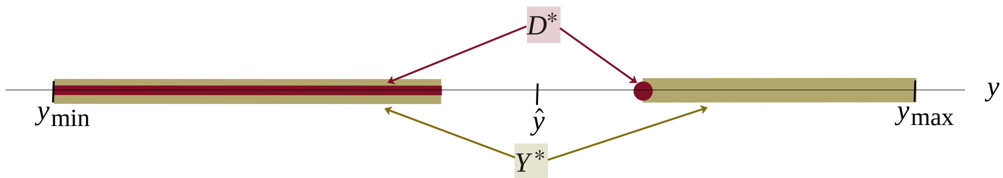

Figure 2: Equilibrium awareness and delegation. The yellow area represents a typical example of equilibrium awareness set Y^* when the principal has limited awareness. The red subset represents the resulting optimal delegation set. When $\hat{y} \notin Y^P$, the principal will be kept unaware of an interval of actions around $\hat{y}$, the cap in the optimal delegation set under full awareness. This way the agent will be allowed to choose the action represented by the red bullet to the right of $\hat{y}$, an action that would be excluded from the delegation set under full awareness.

To prove the statement of the proposition, we need to solve the agent's problem of choosing an awareness set from all subsets of Y^A that include the principal's initial awareness set Y^P. Given that we impose very little structure on Y^P, this optimization problem seems rather complicated. We show, however, that by deriving the properties of the principal's delegation choice for arbitrary awareness sets, we can conveniently restrict our attention to a very simple class of awareness sets from which the agent optimally chooses. We thus proceed recursively: first, we build on the existing literature on optimal delegation and derive the principal's optimal delegation set for an arbitrary awareness set Y; second, with the solution to this problem, we can turn our focus to the main interest of this paper, the agent's choice of the awareness set.

3.3 Delegation Choice

Let $D^*(Y)$ denote the optimal delegation set when the awareness set is Y. We can show the following.

Proposition 3. *Let Assumption 1 be satisfied. Let* $\hat{y}_Y = \arg\min_{y \in Y} |y - \hat{y}|$ *be the element* *of Y that is closest to $\hat{y}$. The optimal delegation for awareness Y is*

$$D^*(Y) = \{y \in Y : y \leq \hat{y}_Y\}.$$

Proof. See Appendix A.2. □

Proposition 3 shows that several important properties of the optimal delegation set in the standard case extend to the situation where Y is an arbitrary subset of $\mathbb{R}$. First, the principal has no incentives to restrict the agent's choice from below. Given that the agent is biased upwards, whenever he prefers $\min Y$ over some other action in the delegation set, so does the principal.

Next, the optimal delegation set $D^*(Y)$ has no "holes" with respect to Y. This result builds on Alonso and Matouschek (2008), who derive conditions under which, in the benchmark case of full awareness, the optimal delegation set is an interval and therefore has no gaps. As we show in the Appendix, their argument perfectly generalizes to generic sets Y that may be non-connected.[12] To gain some intuition, suppose the delegation set includes three actions, y_1, y_2, y_3 with $y_1 < y_2 < y_3$, and consider removing the intermediate action y_2. Then there is an interval of states where the agent switches from y_2 to the low action y_1 and an interval of states where he switches from y_2 to the high action y_3. Given that the principal prefers a lower action than the agent, the first switch benefits the principal but the second one does not, so the question is which effect prevails. The property that the cost of moving away from the bliss point is convexly increasing implies that the cost of the agent switching to y_3 outweighs the gain of the agent switching to y_1, provided that the relative probability mass on the latter event is not too large. The regularity condition on the state distribution assures that this is indeed the case.

It then remains to determine the upper bound of the optimal delegation set. Here Proposition 3 shows that the optimal upper bound is given by the element of Y that is closest to $\hat{y}$. This result has two important implications: first, the optimal delegation set includes all actions belonging to Y that are weakly smaller than $\hat{y}$; second, it includes at most one

[12]In our setting Alonso and Matouschek's (2008) conditions correspond to Assumption 1.

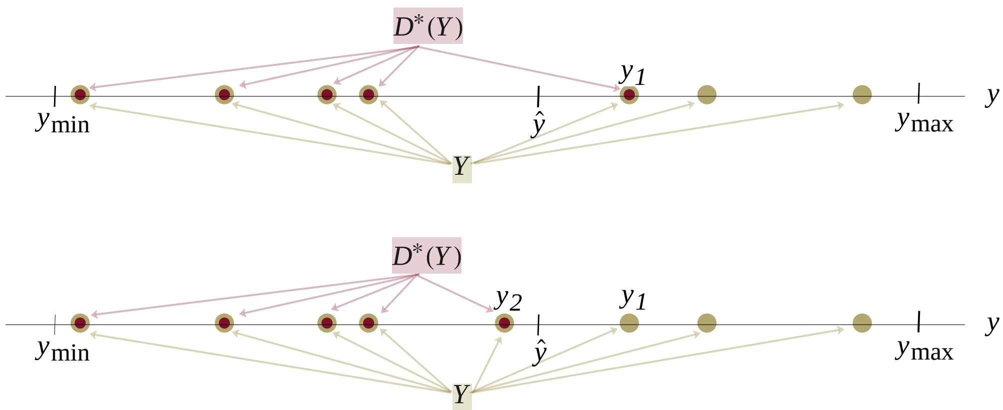

Figure 3: Optimal delegation set $D^*(Y)$. The figures represent two examples of the principal's awareness set Y. In both figures, the yellow bullets represent the set Y while the red bullets represent the resulting optimal delegation set $D^*(Y)$. In the upper figure, the principal includes action y_1 in the delegation set as it is the closest action to $\hat{y}$. In the lower figure, the principal is aware of action y_2 as well and, for this reason, she excludes action y_1 from $D^*(Y)$. Differently put, the awareness of action y_2 by the principal 'crowds out' action y_1 from the resulting delegation set.

action strictly greater than $\hat{y}$. The optimal delegation set under partial awareness can thus be seen as the closest approximation of the optimal interval under full awareness, $[y_{min}, \hat{y}]$, that is available to the principal given her restricted awareness. This approximation includes an element $y > \hat{y}$ if and only if y is closer to $\hat{y}$ than any element of Y smaller than $\hat{y}$. For a graphical illustration see Figure 3.

3.4 Awareness Choice

We can now turn our attention to the agent's optimal strategy of expanding the principal's awareness. As a first observation, notice that if the principal is aware of the threshold action $\hat{y}$, the agent optimally reveals all other actions. Since there is no action closer to $\hat{y}$ than $\hat{y}$ itself, the upper bound of the optimal delegation set will always be $\hat{y}$. Disclosing actions above $\hat{y}$ is thereby irrelevant; the principal will never allow the agent to implement any of them. On the other hand, revealing actions below the threshold $\hat{y}$ is strictly optimal since they will be included in the optimal delegation set, therefore expanding the agent's choice.

Starting now from an arbitrary set Y^P, the above argument implies that the optimal awareness set Y^* is such that the upper bound of the corresponding delegation set $D^*(Y^*)$ is at least $\hat{y}$. Moreover, the only reason for the agent to leave the principal unaware of certain actions is to induce the principal to permit some action strictly greater $\hat{y}$. By Proposition 3 this is optimal for the principal if and only if the principal is not aware of any action closer to $\hat{y}$. Letting $\hat{y}+\Delta, \Delta \geq 0$ denote the upper bound of the induced delegation set, we thus require $(\hat{y}-\Delta, \hat{y}+\Delta) \cap Y = \emptyset$. At the same time, revealing actions below $\hat{y}-\Delta$ and above $\hat{y}+\Delta$ either does not affect the induced delegation set or strictly expands it. It follows that the optimal awareness set is of the form

$$Y^* = [y_{min}, \hat{y}-\Delta] \cup [\hat{y}+\Delta, y_{max}],$$

with the corresponding delegation set

$$D^*(Y^*) = [y_{min}, \hat{y}-\Delta] \cup \{\hat{y}+\Delta\}.$$

The agent is thus permitted to choose from an interval of actions strictly to the left of the full awareness threshold $\hat{y}$ and one action to the right. Given such delegation set, the agent's optimal policy is as follows. In states below $\hat{y}-\Delta$ the agent uses his flexibility and implements his preferred action $y=\theta$. In states above $\hat{y}-\Delta$ the preferred action is not available, so the agent chooses the one closest to his bliss point. For states in the interval $(\hat{y}-\Delta, \hat{y})$ this is the action $\{\hat{y}-\Delta\}$, for the remaining states it is $\{\hat{y}+\Delta\}$. The agent's optimal policy can thus be sumarized by

$$y^*(\theta;\Delta) = \begin{cases} \theta & \text{if} \quad \theta \leq \hat{y}-\Delta \\ \hat{y}-\Delta & \text{if} \quad \hat{y}-\Delta < \theta < \hat{y} \\ \hat{y}+\Delta & \text{if} \quad \theta \geq \hat{y}. \end{cases}$$

Taken together, the previous analysis provides us with a very simple description of the class of delegation and awareness sets that are candidates for an equilibrium in our environment: when deciding which actions to reveal to the principal, the agent implicitly chooses an awareness gap, parametrized by Δ.

To complete the proof of Proposition 2 it remains to show that whenever a gap is feasible, it is also optimal. We can find the optimal awareness gap by considering the agent's reduced form problem of choosing Δ. The feasible values of Δ are determined by the initial level of awareness of the principal Y^P. In particular, the implementable values of Δ are weakly smaller than $\bar{\Delta} := \min_{y \in Y^P} |y - \hat{y}|$, the distance between the action in the principal's awareness closest to $\hat{y}$ and $\hat{y}$. For each Δ in that set, the agent then anticipates the principal's optimal delegation choice and his own optimal policy. Substituting $y^*(\theta; \Delta)$ into the agent's expected payoff, his optimization problem amounts to

$$\max_{\Delta \in [0, \bar{\Delta}]} - \int_{\hat{y}-\Delta}^{\hat{y}} (\hat{y} - \Delta - \theta)^2 dF(\theta) - \int_{\hat{y}}^{1} (\hat{y} + \Delta - \theta)^2 dF(\theta). \tag{2}$$

The following proposition characterizes the solution of this problem.

Proposition 4. *Let Assumptions 1 and 2 be satisfied. The solution of problem* (2) *is given by Min*$[\bar{\Delta}, \Delta^*]$*, where* $\Delta^* > 0$ *solves*

$$\int_{\hat{y}+\Delta^*}^{1} [\theta - (y + \Delta^*)]\, dF(\theta) = \int_{\hat{y}-\Delta^*}^{\hat{y}} [\theta - (y - \Delta^*)]\, dF(\theta) - \int_{\hat{y}}^{\hat{y}+\Delta^*} [\theta - (y + \Delta^*)]\, dF(\theta). \tag{3}$$

Proof. See Appendix A.3. □

The proof of Proposition 4 shows that the agent's payoff as a function of Δ is strictly concave and attains its maximum at Δ^*, as determined by (3). In (3), the left hand side represents the gain from increasing Δ, while the right hand side represents the cost of such change. For a graphical illustration see Figure 7 in Appendix A.3. In all states $\theta > \hat{y}+\Delta^*$, the agent gains from a marginal increase in the gap because the new action $\hat{y} + \Delta^*$ is uniformly closer to his ideal point. The cost of increasing the gap is the utility loss in the states $[\hat{y} - \Delta^*, \hat{y} + \Delta^*]$, where the agent moves away from his ideal action.

The proposition states that the unconstrained solution Δ^* is strictly positive. This can be easily understood by considering the net effect of increasing the gap at $\Delta = 0$. At $\Delta = 0$, the right hand side equals zero: the marginal cost of moving away from the bliss point at the bliss point is zero. The left had side instead clearly takes a positive value for each Δ.

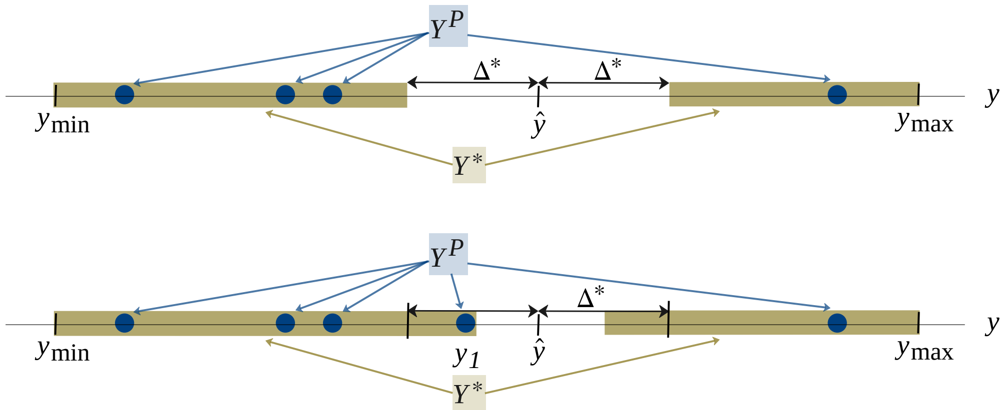

Figure 4: Optimal awareness set Y^*. The figures represent two examples of the principal's initial awareness set Y^P and associated awareness sets $Y^* = Y^P \cup X^*$ after including the information X^* received from the agent. In both figures, the blue bullets represent the set Y^P while the yellow set represents the resulting optimal awareness set Y^*. In the upper figure, the agent keeps the principal unaware of the interval $(\hat{y} - \Delta^*, \hat{y} + \Delta^*)$. In the lower figure, the principal is also aware of action y_1 and, for this reason, the agent finds it optimal to increase the principal's awareness.

Proposition 5. *Let $\Delta^*(\beta)$ be the unrestricted solution to problem (2), as described in Proposition 4, when the principal's preferences parameter is $\beta \geq 0$. Then $\Delta^*(\cdot)$ is an increasing function.*

Proof. See Appendix A.4. □

The proposition shows an intuitive result: the larger the divergence between the principal's and the agent's preferred action is, the more actions the agent wants to hide from the principal. The solution Δ^* is implemented whenever the principal's initial awareness does not constrain the agent in his choice of the gap. If, however, the principal is aware of some action in the interval $(\hat{y} - \Delta^*, \hat{y} + \Delta^*)$, the agent's optimal strategy is to simply choose the largest feasible gap, as shown in Figure 4.

4 Renegotiation

In the baseline problem the agent can only reveal additional actions before he learns the state of the world. For some applications, this seems to be a strong assumption, especially in situations where there is a feasible action that makes both parties better off. One interesting question is then how the equilibrium outcome changes if after learning the state of the world the agent can reveal a set of additional actions to the principal, who then decides whether to permit a new action or to stick with the original contract. We thus consider a model where the agent can renegotiate with the principal after learning the state of the world, at least with a certain probability σ. If the agent gets the opportunity to renegotiate and proposes a new action, the principal understands that the agent's choice signals something about the state of the world. In particular, the principal can infer that the agent only reveals an action if its inclusion benefits him. However - due to the principal's limited awareness - she cannot conceive of alternative actions the agent could have disclosed. This implies that the principal cannot learn from particular actions not being disclosed, an important difference to the case of full awareness. We can show the following.

Proposition 6. *Let Assumptions 1 and 2 be satisfied and suppose the agent can renegotiate with probability* $\sigma \in [0,1]$ *after learning the state of the world. There exists an equilibrium, parametrised by* $\Delta \in [\min\{\Delta^*, \bar{\Delta}\}, \bar{\Delta}]$*, with the following properties:*

- *before learning the state, the agent reveals actions that do not belong to the set* $(\hat{y} - \Delta, \hat{y} + \Delta)$ *and the principal chooses delegation set* $[y_{min}, \hat{y} - \Delta] \cup \{\hat{y} + \Delta\}$*;*
- *if* θ *does not belong to* $(\hat{y} - \Delta, \hat{y} + \Delta)$*, the agent does not renegotiate an takes his preferred action in the delegation set;*
- *if* θ *belongs to* $(\hat{y} - \Delta, \hat{y} + \Delta)$*, the agent proposes his preferred action* $y = \theta$*, which is then permitted by the principal;*

The equilibrium parameter Δ *as a function of* σ *is weakly increasing on* $[0,1]$ *with range* $[\min\{\Delta^*, \bar{\Delta}\}, \bar{\Delta}]$.

Proof. See Appendix A.5. □

Proposition 6 shows that, also when there is renegotiation, there is an equilibrium in which it is optimal for the agent to leave the principal unaware of a gap around $\hat{y}$, parametrised by Δ. Due to her limited awareness, the principal does not expect the agent to reveal other actions in the future and therefore chooses the delegation set $D = [y_{min}, \hat{y} - \Delta] \cup \{\hat{y} + \Delta\}$, as before. If the agent observes the realized state to fall into the gap, he additionally reveals (and recommends) his preferred action $y = \theta$. The principal infers that the agent prefers y over all other actions in the original delegation set and needs to decide whether, conditional on the agent preferring y over any other element in D, she prefers y over the action in D which the agent implements if not permitted y. The proof of Proposition 6 shows that the principal prefers to permit action y if and only if the principal would have permitted y initially, had she been aware of y. Indeed, when choosing the initial delegation set, the principal evaluates the profitability of including a given action only on those states where the action is preferred by the agent. From Proposition 3 we then know that permitting y is optimal for the principal when there is no other action closer to $\hat{y}$. Hence, when the agent proposes an additional action which falls into the gap of the initial delegation set, it is optimal for the principal to implement the new action rather than letting the agent choose from the original delegation set. It follows that conditional on having the chance to renegotiate, the agent is able to implement any action below $\hat{y} + \Delta$.

Unawareness and dynamics. The model with renegotiation highlights two important aspects of games with limited awareness. The first concerns the dynamics of unawareness. Much like information, unawareness is not reversible. This means that if a player becomes aware of an action today, he remains aware of that action in the future (similarly for outcomes, events, etc.). Hence, the more a player reveals at an early stage of the game, the smaller is the collection of awareness sets from which he can choose later on. When there is uncertainty about the future, this creates incentives to hide feasible actions from the other player until later stages of the game. In our model, this feature of unawareness is reflected by the fact that, whenever feasible, the optimal gap of the initial awareness set is strictly larger when there is a positive probability of renegotiation compared to when that probability is zero. The optimal gap without renegotiation is parametrised by $\min\{\Delta^*, \bar{\Delta}\}$, so a larger gap is feasible if and only if $\Delta^* < \bar{\Delta}$. Indeed, if the probability of renegotiation is one, it is optimal for the agent to keep the principal unaware of the largest feasible gap, parameterised by $\bar{\Delta}$.[13] Otherwise the agent compromises between choosing the optimal on-shot awareness set and retaining flexibility in the future by choosing an awareness gap between Δ^* and $\bar{\Delta}$. The more weight the agent assigns to the option of renegotiation in the future, the larger is the initial gap of actions he hides from the principal .

Unawareness and strategic information transmission. In the renegotiation stage principal and agent play a signalling game. One striking feature of the equilibrium described in Proposition 6 is that in the interval $(\hat{y} - \Delta, \hat{y} + \Delta)$ the implemented equilibrium action increases smoothly with the realized state of the world. In other words, there is no pooling of types below the threshold $\hat{y}+\Delta$. This would not be possible under full awareness because, in a candidate equilibrium where types below $\hat{y}+\Delta$ would separate themselves through their announcement, the principal would learn the state of the world and deviate to a strictly lower action. On the other hand, in the case of limited awareness, the principal cannot contemplate moves of the agent she is unaware of and this limits the extent to which he can infer information from the choice of the agent. In particular, if the state is θ and the agent proposes $y = \theta$, the subjective game tree that represents the principal's frame of mind after updating does not include moves of the agent involving any action $a' \in (\hat{y} - \Delta, \hat{y} + \Delta)$ with $a' \neq a$. As a consequence, the principal cannot conceive of the fact that she would have permitted action a' if the agent had proposed a' instead. In the subjective game representing the principal's frame of mind after receiving proposal a, we then have an equilibrium where the agent reveals a in all states that are closer to a than to any other element of the initial delegation set and, conditional on that information, the principal indeed prefers action a.

The fact that limited awareness can reduce the amount of information transmitted by the informed party - and thereby change the equilibrium outcome - extends to other signalling settings. As an example, consider the canonical Crawford and Sobel (1982) cheap talk model,

[13] For this particular case, there is in fact an outcome equivalent equilibrium where the agent reveals nothing in the first stage.

where a biased sender transmits a message to a receiver, who conditionally on the information received, implements his preferred action. In our framework this amounts to dropping the maintained assumption that the principal (the receiver) can commit to a delegation set from which the agent (the sender) can choose. An equilibrium of the cheap talk model under full awareness can be described by a finite set of action Y^{FA} such that each type of agent recommends the action in Y^{FA} that is closest to the realized state of the world and the principal follows the recommendation of the agent (see Crawford and Sobel, 1982). Under partial awareness the equilibrium outcome can be quite different. To illustrate this, suppose for simplicity that the principal is initially aware of a single action y_P and let $\bar{y} = \hat{y} + |\hat{y} - y_P|$. After the agent reveals some action $y \neq y_P$, the principal's frame of mind is described by a partial game which includes the agent's choice of revealing or not revealing y but does not include those subtrees that follow a different revelation of the agent. In this game with partial awareness, there is an equilibrium where the agent proposes his preferred action in every state of the world, the principal believes that the agent proposes a new action if and only if the agent prefers that action over y_P, and the principal follows the agent's proposal if and only if the proposed action is smaller than $\bar{y}$. This is illustrated in Figure 5. It is easy to verify that these actions and believes constitute an equilibrium in the subjective games of each of the two players. Hence, whereas under full awareness each type of agent must be pooled with other types belonging to the same subinterval, under partial awareness this does not have to be the case.

It is interesting to point out that in the described example limited awareness of the principal can benefit both the agent and the principal. To see this, let $\max Y^{FA}$ denote the largest action that can be implemented in an equilibrium under full awareness and define $\bar{y}^{FA} := \hat{y} + |\max Y^{FA} - \hat{y}|$ to be the largest action with the same distance to $\hat{y}$ as $\max Y^{FA}$. Suppose then that the principal is only aware of the singleton $\{\max Y^{FA}\}$. As we just argued, there is a partial awareness equilibrium, where the principal takes the agent's conditional preferred action if the realized state is below $\bar{y}^{FA}$ and takes action $\bar{y}^{FA}$ otherwise. Clearly, the agent prefers this equilibrium outcome over the one under full awareness. With regard

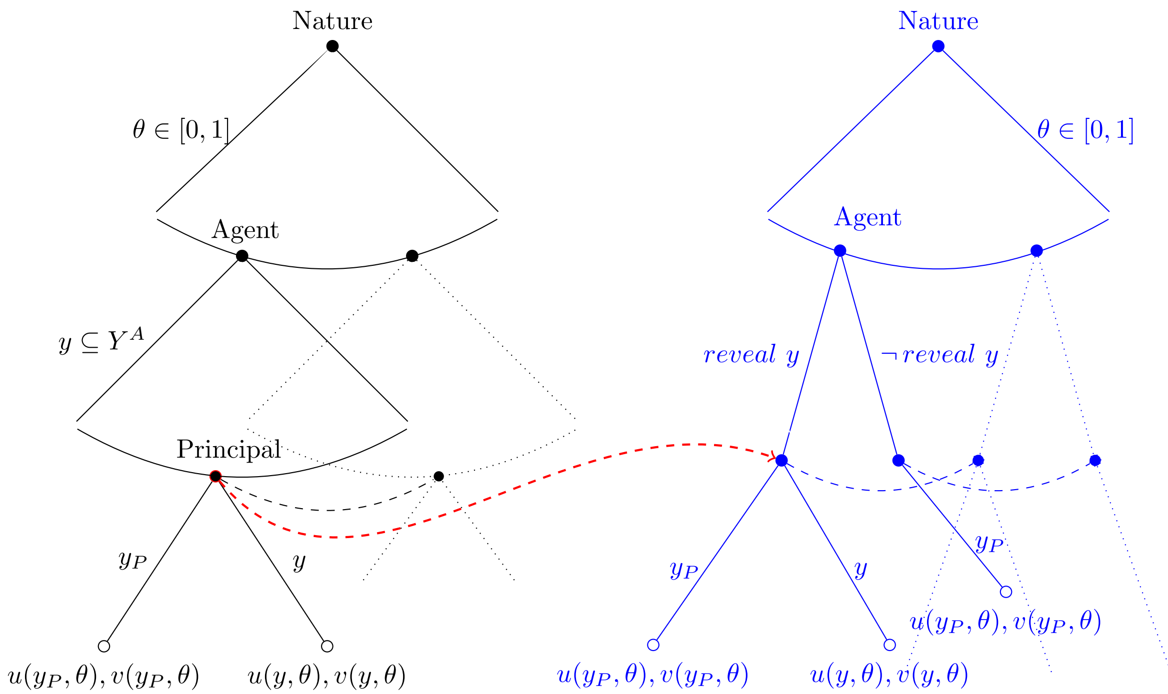

Figure 5: The left game tree represents the modeller's view of a cheap talk game with the principal's initial awareness set being $\{y_P\}$. The left game tree represents the principal's view of the game induced by the agent revealing action y (red dot)

to the principal, Proposition 3 showed that, given the principal's awareness, her optimal delegation set includes all actions weakly smaller than the action closest to $\hat{y}$. This implies that from an ex-ante point of view the principal is indifferent between delegation sets Y^{FA} and $Y^{FA} \cup \{\bar{y}^{FA}\}$. Moreover, the proof of Proposition 3 shows that if the delegation set includes a low and a high action, the principal's payoff can be increased by adding actions that lie in between these two actions. Hence, the principal prefers the delegation interval $[y_{min}, \bar{y}^{FA}]$ to the discrete set $Y^{FA} \cup \{\bar{y}^{FA}\}$. Taken together, this implies that the principal's ex-ante expected payoff from the agent's choice associated to the delegation set $[y_{min}, \bar{y}^{FA}]$ is greater than that associated to Y^{FA}. Since these choices correspond, respectively, to the principal's equilibrium choices in the setting with partial and full awareness, it follows that the principal can benefit from her own unawareness.

5 Discussion

Set of feasible actions. In the baseline model we assumed that the set of available actions is $Y^A = [y_{min}, y_{max}]$. A possible concern is that the principal, despite being unaware of certain elements of Y^A, might understand that Y^A is an interval. The principal could then attempt to include actions outside her awareness, maybe through a description of the properties of such actions. However, as we suggested in the beginning, our analysis applies when Y^A is an arbitrary subset of $\mathbb{R}$, e.g. a finite set, so that *a priori* there is no specific structure of the set of available actions - or simply the awareness set of the agent - that might be commonly known.

To see this, assume Y^A is an arbitrary closed subset of $\mathbb{R}$. The analysis of the optimal delegation set for a given awareness set $Y \subseteq Y^A$ in Section 3.3 remains valid, so we have $D^*(Y) = \{y \in Y : y \leq \hat{y}_Y\}$ (recall that $\hat{y}_Y$ is the element of Y that is closest to $\hat{y}$). With regard to the optimal awareness set, we can first notice that if the agent reveals some $y \in Y^A$, he also reveals all those actions that have a greater distance to $\hat{y}$ than y: their inclusion will weakly expand the agent's choice set. This implies that the optimal awareness set can again be described by a gap Δ and takes the form

$$Y^* = \{y \in Y^A : |y - \hat{y}| \geq \Delta\} \quad \text{with} \quad 0 \leq \Delta \leq \bar{\Delta},$$

where $\bar{\Delta} = \min_{y \in Y^P} |y - \hat{y}|$, as above.

Whether or not the agent reveals all feasible actions to the principal depends on the particular form of Y^A and the principal's initial awareness Y^P. A sufficient condition for full awareness is $\hat{y}_{Y^P} = \hat{y}_{Y^A}$, i.e. the principal is aware of the action that is closest to $\hat{y}$. When this is not the case, the agent leaves the principal unaware of intermediate actions, provided they are close enough to $\hat{y}$ and that there exists a greater action than $\hat{y}_{Y^A}$ that is implementable given the principal's initial awareness.

Quadratic loss preferences and constant bias. While the utility functions we consider are rather special, we should note that the main result of our model - the fact that the agent has an incentive to leave the principal unaware of a set of actions around the optimal threshold under full awareness - remains valid more generally: as long as the principal's and agent's preferences are represented by smooth, single-peaked utility functions that have the property that the ideal action is strictly monotonic in the realized state of the world, the agent's incentives to leave an awareness gap are much the same as in the baseline model. More specifically, imposing an appropriate regularity condition on the state distribution, if the agent is upward biased, the optimal delegation set under full awareness will again be an interval with some upper bound $\hat{y}$.[14] Since the principal cares about the agent's information, the agent can then find some awareness gap around $\hat{y}$ (not necessarily symmetric) such that the principal optimally permits a action greater than $\hat{y}$, provided that $\hat{y} \notin Y^P$. Under the assumption that the agent's utility function is differentiable, we can then replicate the argument following Proposition 4: given that the marginal cost of moving away from the bliss point at the bliss point is equal zero, the net benefit of introducing a marginal gap around $\hat{y}$ will be strictly positive.

To see more concretely how our results generalize to a larger set of models, suppose the agent's and principal's preferences are as follows:[15]

$$u(y, \theta) = y\theta - C(y) \quad \text{and} \quad v(y, \theta) = y(\theta - \beta) - C(y),$$

with $C(\cdot)$ strictly convex and twice differentiable. For $C(y) = \frac{1}{2}y^2$ we are back to the quadratic case. It is easy to show that - under the same Assumption 1 - the optimal delegation set is an interval of the form $[y_{min}, \hat{y}]$, where $\hat{y} = g(\hat{\theta})$, $\hat{\theta}$ solves $\hat{\theta} = \mathbb{E}[\theta - \beta | \theta \geq \hat{\theta}]$, and g is

[14]The optimal delegation literature provides conditions that make interval delegation optimal for a considerably larger class of environments. For the most general treatment see Amador and Bagwell (2013).

[15]Monotone increasing transformations of u and affine transformations of v can easily be allowed as well without any additional complication. State dependent bias can also be introduced with the preferences:

$$u(y, \theta) = y\theta - C(y) \quad \text{and} \quad v(y, \theta) = y(\theta - \beta(\theta)) - C(y),$$

with $\beta'(\theta) \in [0, 1]$. In this case, the analog to the first condition in Assumption 1, which implies interval delegation, is

$$\beta(\theta)f'(\theta) + (1 - \beta'(\theta))f(\theta) > 0.$$

such that $C'(g(\theta)) = \theta$ for all θ. In this case, when the principal has limited awareness, the optimal unawareness interval might be non-symmetric around $\hat{y}$. Following the same line of proof of Proposition 3 in Appendix A.2, we can easily show that the class of sets of actions the principal is left unaware of can be described by the following intervals

$$[\hat{y} - \Delta_1, \hat{y} + \Delta_2], \qquad \text{where} \qquad \frac{C(\hat{y} + \Delta_2) - C(\hat{y} - \Delta_1)}{\Delta_1 + \Delta_2} = C'(\hat{y}).$$

The right had condition above delivers a unique Δ_1 for each Δ_2 and vice versa. The agent optimization problem can therefore be stated as a *function of only one variable* and the solution satisfies the analog to condition (3) in Proposition 4.

Private awareness. For many situations the assumption that the agent knows the set of actions the principal is initially aware of seems rather strong. For instance, a financial intermediary may not know which investment options an investor has encountered before talking to the intermediary. We can show, however, that also in the situation where the agent is not sure about the principal's initial awareness set, the agent optimally hides an interval of actions around the upper cap of the optimal full awareness delegation set.

Proposition 7. *Let the agent's belief about the principal's awareness set Y_P be described by a probability distribution over the set of subsets of $[y_{min}, y_{max}]$. The set of actions the agent the agent optimally reveals takes the form*

$$Y^* = [y_{min}, \hat{y} - \Delta] \cup [\hat{y} + \Delta],$$

for some $\Delta \leq \bar{\Delta}$.

Proof. See Appendix A.6. □

The proof of Proposition 7 shows that the agent can improve on any set of actions by disclosing all actions that have a weakly greater distance to $\hat{y}$ than the action in the initial set closest to $\hat{y}$. The reason is that, no matter what the realized awareness set of the principal is, actions that are further away from $\hat{y}$ do not crowd out any additional actions. The optimal size of the awareness gap for the agent is determined by his beliefs about the principal's initial awareness. A sufficient condition for the agent to hide intermediate actions in equilibrium is that the support of the agent's belief about the principal's initial awareness includes no sets that contain $\hat{y}$. If, on the other hand, the agent believes that it is possible that the principal is aware of $\hat{y}$ and if the probability the agent assigns to this event is sufficiently large, making the principal fully aware might be optimal.

Contingent transfers. The optimal delegation problem differs from the usual contract design problem in that message-contingent transfers are not feasible. We conjecture that, even if they are in fact available, unawareness would still matter. Morgan and Krishna (2008) analyze our setting with full awareness for the case where the principal can offer message-contingent transfers t. They assume that the agent is protected by limited liability so that $t \geq 0$ and that preferences of both contracting parties are quasi-linear. In this case the agent's and principal's payoff function are, respectively, given by[16]

$$u(y, \theta) = -(y - \theta)^2 + t \quad \text{and} \quad v(y, \theta) = -(y - (\theta - \beta))^2 - t.$$

Morgan and Krishna (2008) show that under the optimal contract the implemented action y is non-decreasing in the realized state θ and constant on some interval $[z, 1]$. As before, z can be interpreted as a cap above which no action is permitted. Transfers, on the other hand, are non-increasing in θ and equal to zero on the interval $[z, 1]$ (see Morgan and Krishna (2008), Proposition 1). For the case when f is the uniform distribution they show that, provided the bias is not too large, there is an interval of low states, $[0, z')$, bounded away from z, where the principal pays a positive transfer and the implemented action lies strictly between the principal's and the agent's preferred action. As θ increases, the implemented action in this region increasingly tilts in favor of the agent, until it reaches the agent's ideal point and the transfer is zero. In the region $[z', z]$ the agent simply chooses his preferred action without receiving any transfers, while in the region $[z, 1]$ the agent optimally selects z. Apart from the

[16] More precisely, Morgan and Krishna (2008) assume that the principal's bliss point is θ and the agent's bliss point is $\theta + \beta$. In what follows, we adapt their results accordingly.

first region, the optimal contract thus replicates the allocation we obtain in our framework under full awareness. This in turn implies that the motivation for the agent to keep an awareness gap, in this case around z, is still present: by leaving the principal unaware of an interval of actions around z, the agent induces the principal to permit a action strictly greater than z, exactly as in the case when message-contingent transfers are not feasible.

6 Conclusion

We studied the delegation problem between an agent and a principal with limited awareness of the available actions. We showed that the agent finds it optimal to make the principal aware of actions at the extremes, but leaves her unaware of intermediate options. An achievement of this paper is to formulate a flexible model of delegation with limited awareness and derive a number of properties of the optimal solution. Despite the potential complexity of the resulting double delegation problem, the solution found is remarkably simple and can easily be embedded into more complex frameworks. The key insights of the theory are robust to a number of possible extensions, in particular regarding the set of feasible actions, the player's preferences, the mechanism space, and the information structure. Finally, by introducing renegotiation to the baseline model, we illustrate novel features of dynamic games with limited awareness. We show how limited awareness can restrict inference of information on the equilibrium path and argue that, in some situations, this can help to achieve better outcomes compared to the case when all parties are fully aware. We believe that a further study of such games is a promising task for future research.

Appendix

A Proofs

For the proofs of the following results it is useful to introduce the terms

$$T(y) := F(y)\left(y - \mathbb{E}[\theta - \beta|\theta \le y]\right),$$

and

$$S(y) := (1 - F(y))\left(y - \mathbb{E}[\theta - \beta|\theta \ge y]\right),$$

in the literature referred to as, respectively, backward bias and forward bias (see Alonso and Matouschek, 2008). By Assumption 1 we have

$$T''(y) = \beta f'(y) + f(y) > 0 \quad \text{and} \quad S''(y) = -(\beta f'(y) + f(y)) < 0 \quad \text{for all } y \in [0, 1]$$

Note first that - since $\beta > 0$ - we have $T(y) \ge 0$ for all $y \in [y_{min}, y_{max}]$ and $T(y) > 0$ for $y \ge 0$. The variable S may change sign. Noticing however, that $S(\hat{y}) = S(1) = 0$, strict concavity of S implies that $S(y) > 0$ for all $y \in (\hat{y}, 1)$.

A.1 Proof of Proposition 1

Alonso and Matouschek (2008) show that under Assumption 1 the class of delegation sets are intervals of the form $[y_{min}, \bar{y}]$. We do not repeat their proof here. In Figure 6 we provide a graphical intuitive explanation for the result. In Proposition 3, we consider the case of partial awareness.

Given that we can concentrate on intervals of the mentioned class, the relevant objective function is as follows:

$$-\int_{y_{min}}^{\bar{y}} (\beta)^2 \, dF(\theta) - \int_{\bar{y}}^{1} (\bar{y} - (\theta - \beta))^2 \, dF(\theta).$$

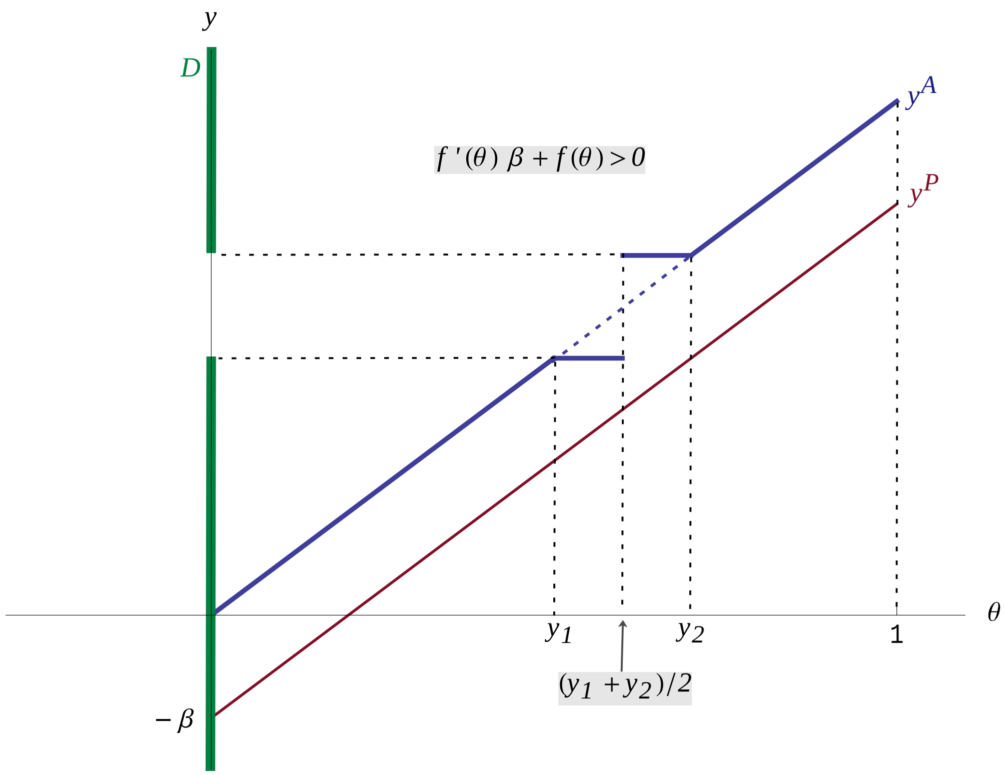

Figure 6: In the horizontal axis the figure reports the set of states, while in the vertical axis it reports the actions. For each θ, the blue 45-degree line represents the preferred action for the agent while the red line reports the preferred actions for the principal. The green set in the vertical axis represents an example of delegation set. Consider two actions y_1 and y_2 with $y_1 < y_2$. If all actions in the interval $[y_1, y_2]$ belong to the delegation set and the realized state θ falls into that interval, the agent chooses his ideal action $y = \theta$. This is represented by the dotted line. If the principal excludes actions (y_1, y_2) from the delegation set, the agent cannot take his preferred action but chooses the one closest to his bliss point. Hence, in states below the midpoint $\frac{y_1+y_2}{2}$ the agent chooses y_1, while in states above the midpoint he chooses y_2. Given that the principal's ideal action lies strictly below the ideal action of the agent, this implies that in states below $\frac{y_1+y_2}{2}$, the implemented action moves closer to the principal's bliss point, whereas in states above $\frac{y_1+y_2}{2}$ it moves further away. Since the cost of moving away from the bliss point is convexly increasing in the distance, the principal's loss outweighs the gain, as long as the probability weight attached to the states below $\frac{y_1+y_2}{2}$ is not too large. The first condition in Assumption 1 - $\beta f'(\theta) + f(\theta) > 0$ - assures that this is indeed the case.

Note that the objective function is continuous and the set Y is compact so a maximum exists. The derivative of the objective function with respect to $\bar{y}$ is:

$$-\bar{y}(1-F(\bar{y}))+\int_{\bar{y}}^{1}(\theta-\beta)dF(\theta). \tag{4}$$

Equation (1) is a simple rearrangement of the first order condition for the principal. The condition $\mathbb{E}[\theta-\beta]>0$ implies that at the point $\bar{y}=0$ the derivative of the objective function is positive. It is easy to see that for $\bar{y}<0$ the derivative of the objective function is larger that at $\bar{y}=0$ so no $\bar{y}\in[y_{min},0)$ can be a solution to the first order condition. The second derivative of the objective function is:

$$\beta f(\bar{y})-(1-F(\bar{y})).$$

Assumption 1 implies that the third derivative of the objective function is positive, implying strict convexity of the first derivative. This implies that the first derivative can cross zero at most at another point. It can be checked by direct inspection of (4) that the other point where the first order condition is satisfied is $\bar{y}=1$. Since $\hat{y}<1$, by convexity of the first derivative, it must be that for $\bar{y}<1$ and close enough to 1 the first derivative is negative. So $\bar{y}=1$ cannot be a maximum. What remains is the interior $\hat{y}$ as claimed.

We now shown the monotonicity of $\hat{y}$ in β. Since a maximum exists, it must be that the following second order necessary condition is satisfied at $\hat{y}(\beta)$ (note we now write explicitly the dependence of $\hat{y}$ on β):

$$\beta f(\hat{y}(\beta))-(1-F(\hat{y}(\beta)))\leq 0. \tag{5}$$

By the strict convexity of the first derivative and the fact that it equals zero from below at $\bar{y}=1$ it must be at $\hat{y}$ the first derivative crosses zero from above at it cannot be flat around $\hat{y}$. Condition (5) must hence be satisfied with strict inequality. We can hence use the implicit function theorem to (4) (Assumption 1 implies that the cumulate F is $\mathcal{C}^1$) to show that $\hat{y}(\beta)$ admits a derivative at each β, which equals:

$$\hat{y}'(\beta) = -\frac{1 - F(\hat{y}(\beta)}{\beta f(\hat{y}(\beta)) - (1 - F(\hat{y}(\beta)))} < 0,$$

where we used the necessary second order condition (5) with strict inequality. Continuous differentiability is guaranteed by the implicit function theorem and can be checked directly in the above expression.

A.2 Proof of Proposition 3

- Statement: consider $y_1, y_2 \in Y$ with $y_1 < y_2$. If $y_1, y_2 \in D^*(Y)$, then all $y \in (Y \cap (y_1, y_2))$ belong to $D^*(Y)$.

  Towards a contradiction suppose there is some $y \in Y$ such that $y \notin D^*(Y)$ and $D^*(Y) \cap [y_{min}, y] \neq \emptyset$, $D^*(Y) \cap [y, y_{max}] \neq \emptyset$. Further, let y^- be the largest element of $D^*(Y)$ strictly smaller than y and let y^+ be the smallest element of $D^*(Y)$ strictly greater than y, that is $y^- = \max\{y' \in D^*(Y) : y' < y\}$ and $y^- = \min\{y' \in D^*(Y) : y' > y\}$. Define $s := \frac{y^- + y^+}{2}$ to be the state at which the agent is indifferent between choosing action y^- and action y^+, and similarly define $r := \frac{y + y^-}{2}$ and $t := \frac{y^+ + y}{2}$ as the states in which the agent is indifferent, respectively, between choosing y^- and y and between y^+ and y.

  Following Alonso and Matouschek (2008), we can write the change in the principal's expected payoff when including action y into the delegation set. The agent changes his choice of action only in states $[r, t]$. In states $[r, s]$ he switches from y^- to y, while in the remaining states $(s, t]$ he switches from y^+ to y. The change in the principal's expected payoff is thus given by

$$\begin{aligned} & -\int_r^t (y-\theta+\beta)^2 f(\theta)\mathrm{d}\theta + \int_r^s (y^- -\theta+\beta)^2 f(\theta)\mathrm{d}\theta + \int_s^t (y^+ -\theta+\beta)^2 f(\theta)\mathrm{d}\theta, \\ = \; & 2(y-y^-)\underbrace{F(r)\left[r-\mathbb{E}\left[\theta-\beta|\theta\le r\right]\right]}_{=T(r)} + 2(y^+-y)\underbrace{F(t)\left[t-\mathbb{E}\left[\theta-\beta|\theta\le t\right]\right]}_{=T(t)} \\ & -2(y^+-y^-)\underbrace{F(s)\left[s-\mathbb{E}\left[\theta-\beta|\theta\le s\right]\right]}_{=T(s)}. \end{aligned}$$

Letting $y = \lambda y^+ + (1-\lambda)y^-$ for some $\lambda \in (0,1)$ so that $y - y^- = \lambda(y^+ - y^-)$, $y^+ - y = (1-\lambda)(y^+ - y^-)$ and $s = \lambda r + (1-\lambda)t$, the payoff difference can be written as

$$2(y^+ - y^-)\left[\lambda T(r) + (1-\lambda)T(t) - T(\lambda r + (1-\lambda)t)\right].$$

From the strict convexity of T, it then follows that the payoff difference is strictly positive. A contradiction.

- Statement: The optimal delegation set satisfies $\min D^*(Y) = \min Y$.

  Consider delegation set D with $\min D(Y) > \min Y$. Letting $y = \min Y$ and $\underline{y} = \min D(\widehat{y})$, the state at which the agent is indifferent between the two actions is given by $s := (y + \underline{y})/2$. If the principal includes y in the delegation set, the agent switches from $\underline{y}$ to y in all states $\theta \le s$. The principal's change in expected payoff when including y is hence given by

$$\begin{aligned} & -\int_0^s (y-\theta+\beta)^2 f(\theta)\mathrm{d}\theta + \int_0^s (\underline{y}-\theta+\beta)^2 f(\theta)\mathrm{d}\theta, \\ = & \int_0^s \left[(\underline{y}-y)(\underline{y}+y) - 2(\underline{y}-y)(\theta-\beta)\right] f(\theta)\mathrm{d}\theta, \\ = & \; 2(\underline{y}-y)T(s), \end{aligned}$$

  which is strictly positive. Including y in the delegation set therefore strictly increases the principal's payoff, which implies $\min D^*(Y) = \min Y$.

- Statement: Let Assumption 1 be satisfied. The optimal delegation set is such that

$$\max D^*(Y) = \arg\min_{y \in Y} |y - \hat{y}|.$$

Consider delegation set D and suppose $\max D < \max Y$. Let $\overline{y} = \max D$ and consider action $y > \overline{y}, y \in Y$. Let $t = \frac{y+\overline{y}}{2}$ denote the state at which the agent is indifferent between the two actions. The change in the principal's payoff when including action y is given by

$$\begin{aligned}
& -\int_t^1 (y - \theta + \beta)^2 f(\theta)\mathrm{d}\theta + \int_t^1 (\overline{y} - \theta + \beta)^2 f(\theta)\mathrm{d}\theta, \\
= & -\int_t^1 \left[(y - \overline{y})(y + \overline{y}) - 2(y - \overline{y})(\theta - \beta)\right] f(\theta)\mathrm{d}\theta, \\
= & -2(y - \overline{y})S\left(t\right).
\end{aligned}$$

This change is weakly positive if and only if $S(t) \leq 0$ and hence if and only if $t \leq \hat{y}$. Since t is the midpoint of $\overline{y}$ and y, this condition holds if and only if the distance between $\overline{y}$ and $\hat{y}$ is weakly greater than the distance between y and $\hat{y}$, i.e. $|\overline{y} - \hat{y}| \geq |y - \hat{y}|$.

□

A.3 Proof of Proposition 4

Let the agent's payoff as a function of Δ be defined by (recall that for $\theta \leq \hat{y}$ the payoff equal is zero):

$$U(\Delta) = -\int_{\hat{y}-\Delta}^{\hat{y}} (\hat{y} - \Delta - \theta)^2 f(\theta)\mathrm{d}\theta - \int_{\hat{y}}^1 (\hat{y} + \Delta - \theta)^2 f(\theta)\mathrm{d}\theta. \tag{6}$$

The first and second derivative of $U(\Delta)$ are

$$\frac{\mathrm{d}U(\Delta)}{\mathrm{d}\Delta} = 2\int_{\hat{y}-\Delta}^{\hat{y}} \left[\hat{y} - \Delta - \theta\right] f(\theta)\mathrm{d}\theta - 2\int_{\hat{y}}^1 \left[\hat{y} + \Delta - \theta\right] f(\theta)\mathrm{d}\theta, \tag{7}$$

$$\frac{\mathrm{d}^2U(\Delta)}{\mathrm{d}\Delta^2} = -2[1 - F(\hat{y} - \Delta)] < 0. \tag{8}$$

The function $U(\Delta)$ is strictly concave in Δ and hence has a unique solution on $[0, \bar{\Delta}(Y)]$. The interior solution of the agent's optimization problem, Δ^*, is characterized by the first-order condition that equalizes the expression (7) to zero. The expression in (3) is obtained after simple rearrangements of the terms in (7).

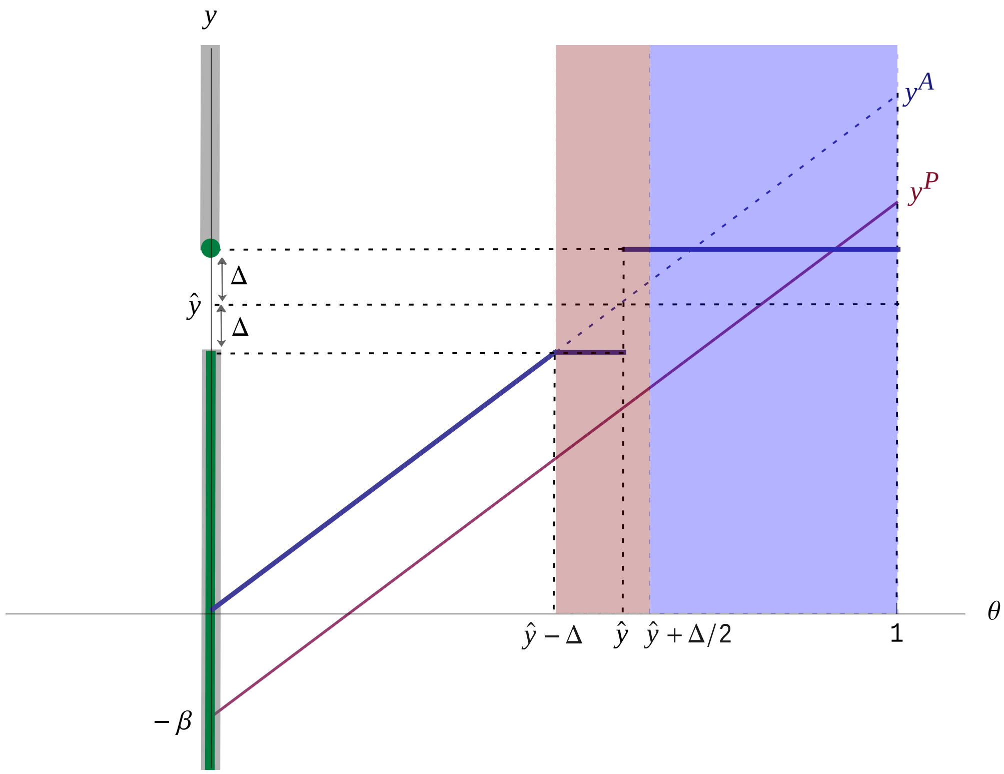

Figure 7: In the horizontal axis the figure reports the set of states, while in the vertical axis it reports the feasible actions. The green set in the vertical axis represents a typical equilibrium delegation set. For each θ, the blue 45-degree line represents the preferred action for the agent while the red line reports the preferred actions for the principal. For states from $\hat{y} + \frac{\Delta}{2}$ the agent gains as the new pooling action is uniformly closer to his ideal point compared to $\hat{y}$. This is represented by the blue area in the figure. The cost of increasing the gap is the utility loss in the states $\left[\hat{y} - \Delta, \hat{y} + \frac{\Delta}{2}\right]$, where the agent moves away from his ideal action. The states implying a loss compared to $\hat{y}$ are represented by the red area. Note that for $\Delta \approx 0$ the red area vanishes. Since at $\hat{y}$ the agent chooses his bliss point action, by increasing the gap he enjoys first order gains while losses are zero to the first order. The unrestricted optimal Δ^* equalizes marginal gains with marginal losses maximizing the overall net gain.

A.4 Proof of Proposition 5

First, let us write the agent's payoff as a function of Δ and the parameter β:

$$U(\Delta;\beta) = -\int_{\hat{y}(\beta)-\Delta}^{\hat{y}} (\hat{y}(\beta)-\Delta-\theta)^2 f(\theta)\mathrm{d}\theta - \int_{\hat{y}(\beta)}^{1} (\hat{y}(\beta)+\Delta-\theta)^2 f(\theta)\mathrm{d}\theta.$$

Recall, the solution to the problem solves

$$\int_{\hat{y}(\beta)-\Delta^*}^{\hat{y}(\beta)} \left[\hat{y}(\beta)-\Delta^*-\theta\right] f(\theta)\mathrm{d}\theta - \int_{y(\hat{\beta})}^{1} \left[\hat{y}(\beta)+\Delta^*-\theta\right] f(\theta)\mathrm{d}\theta = 0.$$

Since F is $\mathcal{C}^1$ by Assumption 1, $\hat{y}(\beta)$ is $\mathcal{C}^1$ from 1, and $U''_{\Delta,\Delta} < 0$, the conditions for applying the implicit function theorem are satisfied. There is hence a function $\Delta^*(\beta)$ describing the unrestricted solution for the agent that solves the first order condition: $U'_{\Delta}(\Delta^*(\beta);\beta) = 0$, which becomes an identity when seen as a function of β, and:

$$\Delta^{*\prime}(\beta) = -\frac{U''_{\Delta,\beta}(\Delta^*(\beta);\beta)}{U''_{\Delta,\Delta}(\Delta^*(\beta);\beta)}.$$

The statement in the proposition will hence be shown if we can prove that $U''_{\Delta,\beta}(\Delta^*(\beta);\beta) > 0$. Differentiating the expression of the first order condition (3) with respect to β keeping Δ^* as fixed, after some rearrangement, delivers:

$$U''_{\Delta,\beta}(\Delta^*(\beta);\beta) = -\hat{y}'(\beta)\left[1 + F\left(\hat{y}(\beta)-\Delta^*(\beta)\right) - 2F(\hat{y}(\beta))\right].$$

Since $\hat{y}'(\beta) < 0$ from Proposition 1, we would be done if $1+F\left(\hat{y}(\beta)-\Delta^*(\beta)\right)-2F(\hat{y}(\beta)) > 0$. Note that this inequality can be equivalently written as:

$$2(1-F(\hat{y}(\beta))) > 1 - F\left(\hat{y}(\beta)-\Delta^*(\beta)\right).$$

Now, if we use $\hat{y}(\beta) = \mathbb{E}[\theta-\beta|\theta \geq \hat{y}(\beta)]$ and rearrange the first order condition, we obtain:

$$\left[1-F(\hat{y}(\beta)-\Delta^*(\beta))\right]\left[\mathbb{E}[\theta|\theta \geq \hat{y}(\beta)-\Delta^*(\beta)] - (\hat{y}(\beta)-\Delta^*(\beta))\right] = 2[1-F(\hat{y}(\beta))]\beta.$$

Since $\hat{y}(\beta) - \Delta^*(\beta) < \hat{y}(\beta)$ from the definition of $\hat{y}(\beta)$, it must be that

$$\mathbb{E}[\theta|\theta \geq \hat{y}(\beta) - \Delta^*(\beta)] - (\hat{y}(\beta) - \Delta^*(\beta)) > \beta,$$

which implies $(1 - F(\hat{y}(\beta) - \Delta^*)\beta))) < 2(1 - F(\hat{y}(\beta)))$ as desired.

A.5 Proof of Proposition 6

Suppose that after the initial stage of revelation the principal's awareness is Y and the optimal delegation set is D.

- Consider the possibility that after learning the state of the world the agent reveals a new action $\tilde{y} \in (\min D, \max D)$ and let y^- be the greatest element of D smaller than $\tilde{y}$ and y^+ be the smallest element of D greater than $\tilde{y}$. Optimality of D with respect to the principal's awareness Y implies that there are no other actions within the principal's awareness between y^- and y^+. When the agent proposes action $\tilde{y}$, the principal learns that the state lies between $(\tilde{y} + y^-)/2$ and $(\tilde{y} + y^+)/2$. The proof of Lemma 3 shows that conditional on these states, the principal prefers to include the action $\tilde{y}$.

- Next, suppose the agent reveals a new action $\tilde{y} < \min D$. Since D is optimal with respect to Y, there is no element in Y smaller than $\underline{y}$. The fact that the agent proposes action $\tilde{y}$ allows the principal to conclude that $\theta < (\tilde{y} + \underline{y})/2$. Since the principal's preferred action is strictly smaller than the agent's preferred action, this condition assures that the principal prefers $\tilde{y}$ over $\underline{y}$.

- Finally, suppose the agent reveals some action $\tilde{y} > \max D$. Assume first that there are no actions in Y that are strictly greater than $\max D$. In this case the principal learns that $\theta > (\tilde{y} + \max D)/2$. He optimally allows the agent to implement $\tilde{y}$ if

$$\mathbb{E}[-(\max D - \theta + \beta)^2|\theta > (\tilde{y} + \max D)/2] \leq \mathbb{E}[-(\tilde{y} - \theta + \beta)^2|\theta > (\tilde{y} + \max D)/2].$$

This inequality can be rewritten as

$$(\tilde{y} - \max D)\left((\tilde{y} + \max D)/2 - \mathbb{E}[\theta - \beta|\theta > (\tilde{y} + \max D)/2]\right) \leq 0,$$

which is satisfied if and only if $(\tilde{y} + \max D)/2 \leq \hat{y}$, that is, if and only if the distance between $\tilde{y}$ and $\hat{y}$ is weakly smaller than that between $\max D$ and $\hat{y}$.

Now suppose $(\tilde{y} + \max D)/2 \leq \hat{y}$ holds and the principal is aware of some additional action y' greater than $\max D$. We want to show that, upon becoming aware of $\tilde{y}$, the principal prefers to implement $\tilde{y}$ over y'. By optimality of D with respect to Y, the action y' must have a greater distance to $\hat{y}$ than $\max D$. This implies that $\tilde{y}$ is strictly smaller than y' and that conditional on the state being greater than the midpoint $(\tilde{y} + y')/2$, the principal prefers $\tilde{y}$ over y'. But if the lower action $\tilde{y}$ is preferred to y' conditional on $\theta \geq (\tilde{y}+y')/2$, it must also be preferred conditional on $\theta \geq (\tilde{y}+\max D)/2$ (remember $(\tilde{y}+\max D)/2 < (\tilde{y}+y')/2$). Hence, upon learning $\theta \geq (\tilde{y}+\max D)/2$, the principal prefers to delegate $\tilde{y}$ over any other action within his awareness set

. Taken together, this implies the following. Given a delegation set D that is optimal with respect to the principal's awareness after the first updating, Y, the principal permits any action that is smaller than $\max D$ and any action that is larger than $\max D$ as long as its distance to $\hat{y}$ is smaller than that between $\max D$ and $\hat{y}$. Remembering that $\max D$ is the action in Y that is closest to $\hat{y}$, the relevant constraint in this stage of the game is thus imposed by the action in the principal's awareness with the smallest distance to $\hat{y}$. The larger this distance is, the larger is the set of actions the agent is able to implement after learning the state of the world. This implies that at the outset of the game it is optimal for the agent to reveal a set Y in the class of awareness sets parametrised by a gap Δ, as in the game without renegotiation. The optimal value of Δ solves the following optimization problem:

$$\max_{\Delta \leq \bar{\Delta}} \qquad U_\sigma(\Delta) := -\sigma \int_{\hat{y}+\Delta}^{1} (\hat{y} + \Delta - \theta)^2 dF(\theta) + (1-\sigma)U(\Delta).$$

where $U(\Delta)$ is defined by (6). When $\sigma = 0$, we renegotiation plays no role and U_σ is maximized by $\min\{\Delta^*, \bar{\Delta}\}$, as we established in Proposition 4. We will thus consider the case $\sigma > 0$. We have

$$\begin{aligned} U'_\sigma(\Delta) &= -2\sigma \int_{\hat{y}+\Delta}^{1} \underbrace{(\hat{y}+\Delta-\theta)}_{<0} dF(\theta) + (1-\sigma)U'(\Delta), \\ U''_\sigma(\Delta) &= -2\sigma(1-F(\hat{y}+\Delta)) + (1-\sigma)U''(\Delta). \end{aligned}$$

Since $U''(\Delta) < 0$ (see Section A.3), we have $U''_\sigma < 0$, so the problem is strictly concave. The unconstrained solution of the optimization problem is characterized by the first-order condition

$$2\sigma \int_{\hat{y}+\Delta}^{1} (\hat{y}+\Delta-\theta) dF(\theta) = (1-\sigma)U'(\Delta) \tag{9}$$

Let Δ^{**} denote the value of Δ solving condition (9). Since the left-hand-side is strictly negative, (9) requires $U'(\Delta^{**}) < 0$ and hence $\Delta^{**} > \Delta^*$. The agent's optimisation problem is then solved by $\min\{\Delta^{**}, \bar{\Delta}\}$. Finally, it is easy to that $U'_\sigma(\Delta) < U'_{\sigma'}(\Delta)$ for all $\sigma < \sigma'$ and $\Delta > \Delta^*$. Letting $\Delta^{**}(\sigma)$ be the unconstrained solution for the parameter σ, this directly implies $\Delta^* * (\sigma) < \Delta^* * (\sigma')$. Notice that for $\sigma = 1$, we have $U'_\sigma(\Delta) > 0$ for all $\Delta \leq 1 - \hat{y}$, so $\Delta^{**}(1) = \min\{1-\hat{y}, \bar{\Delta}\}$. Given that $U'_\sigma(\Delta)$ is continuous in σ, the statement of Proposition 6 follows.

A.6 Proof of Proposition 7

We want to show that revealing an awareness set of the form $[y_{min}, \hat{y}-\Delta] \cup [\hat{y}+\Delta, y_{max}]$, $\Delta \leq \bar{\Delta}$ is optimal. Towards a contradiction, suppose this is not the case and let the optimal awareness set be denoted by Y. Define $\tilde{\Delta}$ to be the smallest value of Δ such that $\tilde{Y} \subseteq [y_{min}, \hat{y}-\Delta] \cup [\hat{y}+\Delta, y_{max}]$ and $\tilde{Y} = [y_{min}, \hat{y}-\Delta] \cup [\hat{y}+\Delta, y_{max}]$ to be the associated awareness set. Suppose the principal's realized awareness set is Y_P. According to Proposition 3 the induced delegation sets from revealing, respectively, Y and $\tilde{Y}$ are

$$
\begin{aligned}
D^*(Y \cup Y_P) &= \{y \in Y \cup Y_P : y \leq \hat{y}_{Y \cup Y_P}\}, \\
D^*(\tilde{Y} \cup Y_P) &= \{y \in \tilde{Y} \cup Y_P : y \leq \hat{y}_{\tilde{Y} \cup Y_P}\}.
\end{aligned}
$$

In order for Y to yield a strictly higher payoff for the agent than $\tilde{Y}$, there must exist some awareness set Y_P and some action y such that $y \in D^*(Y \cup Y_P)$ and $y \notin D^*(\tilde{Y} \cup Y_P)$. By Proposition 3 we know that $D^*(\tilde{Y} \cup Y_P)$ includes all actions in $\tilde{Y}$ weakly smaller than $\hat{y}$. Given $Y \subseteq \tilde{Y}$, it follows that $y > \hat{y}$. Proposition 3 further shows that the optimal delegation set includes at most one action strictly greater than $\hat{y}$. By definition of $\tilde{\Delta}$, the set Y includes an action whose distance to $\hat{y}$ is $\tilde{\Delta}$. This implies that the largest action in $D^*(Y \cup Y_P)$ is weakly smaller than $\hat{y} + \tilde{\Delta}$. Hence, we have $y \leq \hat{y} + \tilde{\Delta}$. Also, since y belongs to $D^*(Y \cup Y_P)$, it follows that there is no action in Y_P strictly closer to $\hat{y}$ than y. However, the property $|y - \hat{y}| \leq \tilde{\Delta}$, together with the fact that there is no action in Y_P that is closer to $\hat{y}$ than y, implies that y must also belong to $D^*(\tilde{Y} \cup Y_P)$. A contradiction.

□

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